Lecture 3 - Translating Natural Language to Propositional Calculus

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1 Translation Errors

In Lecture 1 we learned that propositions are declarative statements that can be evaluated as either true or false. When translating natural language to propositional logic we have to be careful because

1. Some declarative statements are not propositions.
2. Some statements can be translated in multiple ways because they are ambiguous.
3. We can loose information during translation.

1.1 Sentences That Are Not Propositions

Some statements like textitLet it rain are declarative but are not propositions. For instance we can not say, it is true that *let it ran*. Thus we can not reason anything about it using propostional logic. textitI wished it would rain however is a proposition, that is, I either wished it would rain or I never made that wish.

1.2 Ambiguity

When we translate a sentence into propositional logic formula we need to define what the variables in the formula represent. For example when translating

Jack plays football and Jill plays volleyball or Jill plays softball

we can write

Let p be the proposition *Jack plays football* and let q be the proposition *Jill plays volleyball* and let r be the proposition *Jill plays softball.*

This statement and others are ambiguous. One way to translate this is as $(p \wedge (q \vee r))$. This would be the more common translation. When this formula is true it asserts that both Jack and Jill play a sport; Jack plays football and Jill plays either volleyball or softball.

But we can also interpret the sentence as $((p \wedge q) \vee r)$. When this formula is true it asserts that both Jack and Jill play sports (football and volleyball respectively) or that Jill plays softball.

When given statements that are ambiguous we must stop and ask which meaning is intended. To avoid this we need need to be more clear in our English sentence construction.

1.3 Loss of information

Lets begin with a more clear sentence. Consider *Jack plays football and Jill plays either volleyball or softball.* One valid way to translate this sentence is to simply provide the following definition:

Let p be the proposition *Jack plays football and Jill plays either volleyball or softball.*

This is a valid translation and is acceptable if we don't need to reason about the individual propositions as defined in the earlier translation. If we do, however, this translation is insufficient as we loose information that may be needed later.

Similarly if we're considering the sentence, *Jack plays football or soccer* we can define p as the proposition *Jack plays football or socker.* But if we do this we have to be careful when negating. In this case, the negation, $\neg p$, is *Jack does not play football or socker.* Notice that the negation is not *Jack does not play football and socker.* We don't apply DeMorgan's rule since we are negating a single proposition and not a disjunction of propositions.

2 Indirect Translations

In this section we point out a number of common phrases and their translation.

2.1 But

Many sentences with *but* in them, like *The temperature is not hot, but the sun is shining*, can be interpreted as a *conjunction* of propostions. In the example above example, an equivalent statement is *The temperature is not hot and the sun is shining.* If we let p be the proposition *The temperature is hot* and let q be the proposition *The sun is shining*, then the propsitional translation is $\neg p \wedge q$.

2.2 Affirmative Vs. Non-Affirmative

We often define propositional variables using affirmative propositions, that is, positive statements. For example to translate in *The temperature is not hot* we let p be the proposition *The temperature is hot* and the translation is then, $\neg p$.

But we are not required to define propositional variables in the affirmative. For the sentence *The weather is not hot and the sun is not shining* we can also define p as the proposition *The weather is not hot* and we can define q as the proposition *The sun is not shining.* In this case the propositional translation for the sentence is $p \wedge q$.

2.3 Neither

The word *neither*, as in *The weather is neither hot nor rainy* is also interpreted as a conjunction but here the individual propositions are negated. An equivalent sentence therefore is *The weather is not hot and the weather is not rainy.* If we let p be the proposition *The weather is hot* and let q be the proposition *The weather is rainy*, then the propositional translation for the sentence is $\neg p \wedge \neg q$.

2.4 Implications

Implications can be expressed in many ways that are all equivalent.

- If it is snowing, school is canceled.
- If it is snowing then schools is canceled.
- It is snowing implies school is canceled.
- It is not snowing or school is canceled.
- If school is not canceled then it is not snowing.
- It is snowing only if schools is canceled.

p	q	$p \Rightarrow q$	$(\neg p)$	$(\neg p) \vee q$	$(\neg q)$	$(\neg q) \Rightarrow (\neg p)$
0	0	1	1	1	1	1
0	1	1	1	1	0	1
1	0	0	0	0	1	0
1	1	1	0	1	0	1